SUMMIT MATH

Learn at your **OWN** pace.

ALGEBRA 1

second edition

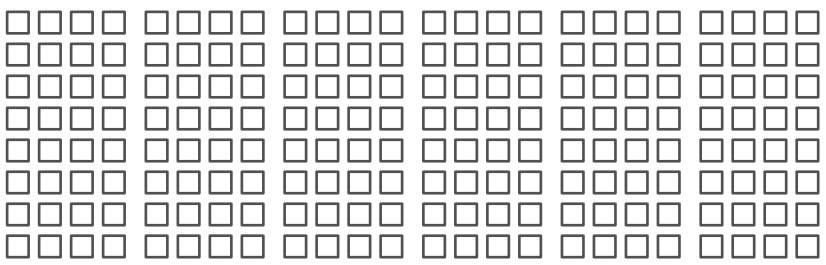
3 PROPERTIES OF EXPONENTS

CONTENTS

Section 1

INTRODUCTION TO EXPONENTS

1. How many squares do you see in the image below?



Counting the squares above is an easier task for an older student than it is for, say, a five-year-old child. To a kindergartener, the likely way to approach the task above is by counting. . . one square at a time. 1, 2, 3, 4, . . ., 27, 28, 29, 30, . . . and so on.

Fortunately, you don't need to count like this any more. At some point, you learned how to speed up the tedious task of counting by combining groups of numbers and counting those groups multiple times: hence the word multiplication. For example, "4 + 4 + 4 + 4 + 4 + 4 + 4 + 4" can be expressed as "8 sets of 4" or "8 times 4". Multiplication is a way to represent repeated addition.

2. Consider an example of another repeated operation. You'll need a spare sheet of paper.

   a. Take a sheet of paper and fold the paper in half once. When you unfold it, the paper will now be divided into how many sections?

   b. Fold the paper in half once again. Now fold that portion in half, and then fold this entire portion in half a third time. When you open it up, the paper will be divided into how many sections?

   c. If you fold a sheet of paper in half 5 times and open it up again, into how many sections will the paper be divided?

3. You should now notice a pattern in the growth in the number of sections as the paper is folded.

   a. A sheet of paper will be divided into 64 sections if you fold it in half ____ times.

   b. A sheet of paper will be divided into 256 sections if you fold it in half ____ times.

4. If you fold a sheet of paper in half *N* times and open it up again, into how many sections will the paper be divided?

If you could fold a sheet of paper in half 10 times and open it up again, it would be divided into $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ sections. If you could fold it in half 75 times and open it up again, it would be divided into. . . as with the squares earlier, this is getting rather tedious again without more condensed notation.

5. Since "3 + 3 + 3 + 3 + 3 + 3 + 3" can be expressed in a more concise form as "3×7", how can you express $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ in a more concise form?

Just as multiplication represents repeated addition, an exponent is a type of notation that is used to represent repeated multiplication. An exponent can be used to write $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ in a much more condensed form as 2^{10} (2 raised to the 10^{th} power).

6. Write each of the following expressions as a single number raised to an exponent.

a. $5\cdot5\cdot5$ b. $10\cdot10\cdot10\cdot10$ c. $(-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)$

7. Write each of the following expressions as a single variable raised to an exponent.

a. $x\cdot x\cdot x$ b. $y\cdot y\cdot y\cdot y$ c. $z\cdot z\cdot z\cdot z\cdot z$

8. Write each of the following expressions as a repeated multiplication expression. For example, 7^3 can be written as $7\cdot7\cdot7$.

a. 10^3 b. $(-2)^4$ c. m^5

9. Write each of the following expressions as a repeated multiplication expression. For example, 7^3 can be written as $7\cdot7\cdot7$.

a. $3^3\cdot3^2$ b. $y^3\cdot y^2\cdot y$

10. How would you write $(2x)^4$ as a repeated multiplication expression?

11. How would you write $(x+1)^2$ as a repeated multiplication expression?

Section 2

MULTIPLYING EXPRESSIONS WITH EXPONENTS

12. Consider the result when expressions with exponents are multiplied together. Simplify each product below and write each result as a single number raised to an exponent.

a. $3^3 \cdot 3^2$ b. $4^2 \cdot 4^6$ c. $5^3 \cdot 5^6$ d. $6^{10} \cdot 6^{10}$

13. Write each expression as a single number raised to an exponent.

a. $3^2 \cdot 3 \cdot 3^4$ b. $(-1)^7 \cdot (-1)^2 \cdot (-1)$ c. $2^x \cdot 2^y$

14. Write each expression as a single variable raised to an exponent.

a. $x^2 \cdot x$ b. $x^2 \cdot x^3$ c. $x^3 \cdot x^5$

15. Write each expression as a single variable raised to an exponent.

a. $x^2 \cdot x \cdot x^4$ b. $y^7 \cdot y^2 \cdot y$ c. $x^A \cdot x^B \cdot x^C$

16. The previous scenarios illustrate The Product Rule, which applies when you multiply like bases and determine the exponent of your result. Write out The Product Rule rule in your own words as if you were explaining it to someone.

17. Consider that $5 \cdot 5 = 25$.

a. Does $5^2 \cdot 5^3$ have the same value as 25^5 or does $5^2 \cdot 5^3$ equal 5^5? How can you persuade someone else that your conclusion is accurate?

b. Does $3^5 \cdot 3^4$ have the same value as 3^9 or does $3^5 \cdot 3^4$ equal 9^9?

Section 4

RAISING AN EXPONENT TO AN EXPONENT

38. Fill in the blanks to write the expression in an expanded form. The first one is done for you.

a. $\left(2^2\right)^3 = 2^2 \cdot 2^2 \cdot 2^2$ b. $\left(2^3\right)^4 =$ _____ c. $\left(2^{10}\right)^5 =$ _____

39. How many 2's are multiplied together to form each expression?

a. $\left(2^2\right)^3$ b. $\left(2^3\right)^4$ c. $\left(2^5\right)^{10}$ d. $\left(2^8\right)^X$

40. How many x's are multiplied together to form each expression?

a. $\left(x^4\right)^2$ b. $\left(x^2\right)^5$ c. $\left(x^6\right)^{11}$ d. $\left(x^4\right)^Y$

41. Rewrite the expression $\left(B^{10}\right)^3$ as a base raised to an exponent.

42. Write each expression below as a repeatedly multiplied expression.

a. $\left(2x\right)^3$ b. $\left(-3x\right)^3$

43. Try to simplify each expression. In this case, "simplify" means to write an equivalent form of the expression shown without parentheses in your expression.

a. $\left(5x^4\right)^2$ b. $\left(-2y\right)^3$ c. $\left(\frac{z^2}{3}\right)^4$

44. How would you write $\left(-3x^{10}\right)^3$ as a repeated multiplication expression?

45. Simplify each expression.

a. $\left(10x^7\right)^2$ b. $\left(-4y^2\right)^3$ c. $\left(-5xy^2\right)^4$

Section 7

NEGATIVE EXPONENTS

65. Recall the patterns you noticed in the previous scenarios to determine the value of each of the following expressions. Express your answer as a fraction.

a. 6^{-1} b. 7^{-1} c. 8^{-1} d. X^{-1}

66. In a previous scenario, you found that $2^2=4$ while $2^{-2}=\frac{1}{4}$. Additionally, $2^3=8$ while $2^{-3}=\frac{1}{8}$.

a. What is the relationship between these pairs of results?

b. Since $2^4=16$, what is the value of 2^{-4}?

c. What is the value of 4^{-3}?

67. How can you find the value of A^X if A is a positive integer and X is a negative integer?

68. You have seen earlier that 3^{-1} has the same value as $\left(\frac{1}{3}\right)^1$. Since 3 can be written as $\frac{3}{1}$, it follows that $\left(\frac{3}{1}\right)^{-1}=\left(\frac{1}{3}\right)^1$. Following this structure, what is the value of $\left(\frac{1}{3}\right)^{-1}$?

69. Determine the value of each of the following expressions.

a. $\left(\frac{2}{3}\right)^{-1}$ b. $\left(\frac{4}{5}\right)^{-1}$ c. $\left(\frac{A}{B}\right)^{-1}$

70. Determine the value of each of the following expressions.

a. $\left(\frac{6}{7}\right)^{-2}$ b. $\left(\frac{9}{8}\right)^{-2}$ c. $\left(\frac{A}{B}\right)^{-2}$

71. What is the value of 0^{-1}?

Section 10
ANSWER KEY

1.	192 squares
2.	a. 2 b. 8 c. 32
3.	a. 6 b. 8
4.	2^N
5.	2^{10}
6.	a. 5^3 b. 10^4 c. $(-3)^5$
7.	a. x^3 b. y^4 c. z^5
8.	a. $10\cdot10\cdot10$ b. $(-2)(-2)(-2)(-2)$ c. $m\cdot m\cdot m\cdot m\cdot m$
9.	a. $3\cdot3\cdot3\cdot3\cdot3$ b. $y\cdot y\cdot y\cdot y\cdot y\cdot y$
10.	$2x\cdot2x\cdot2x\cdot2x$
11.	$(x+1)(x+1)$
12.	a. 3^5 b. 4^8 c. 5^9 d. 6^{20}
13.	a. 3^7 b. $(-1)^{10}$ c. 2^{x+y}
14.	a. x^3 b. x^5 c. x^8
15.	a. x^7 b. y^{10} c. x^{A+B+C}
16.	When multiplying like bases, you can add the exponents
17.	a. $5^2\cdot5^3=5^5$ b. $3^5\cdot3^4=3^9$
18.	a. $10^2\cdot x^2$ b. 4^3y^2
19.	a. 2^66^{10} b. 7^3x^6 c. m^9p^4
20.	a. 3^{2x} b. 8^{2y+5} c. 5^{3x+1}
21.	a. 15 b. x^{10} c. $2x^7$
22.	a. $-4x^3$ b. $21y^{10}$ c. $45x^9$
23.	a. $3w^5\cdot3w^5\rightarrow9w^{10}$ b. $18x^6$ c. $-4m^7\cdot-4m^7\rightarrow16m^{14}$
24.	$\frac{2}{1}$ or just 2
25.	two times
26.	2^3
27.	a. two b. three c. six
28.	a. 3^2 b. 7^2 c. 4^2
29.	a. 3^2 b. 7^2 c. 9^1 or just "9"
30.	a. 2^3 b. 4^1 or just "4" c. 5^0 or just "1"
31.	a. 5^3 b. $(-1)^3$ c. x^3
32.	a. x b. x^3 c. 1 d. x^8 e. x^{A-B}
33.	a. $4x$ b. $\frac{x}{2}$ c. $\frac{10}{x}$ d. $\frac{y}{2}$
34.	a. $\frac{x^4}{2}$ b. $\frac{7x^4y}{6}$ c. $\frac{2}{5}$ d. $\frac{3x^{A-B}}{2}$
35.	$X - Y$
36.	When dividing like bases, simplify the fraction formed by the coefficients and subtract the exponents.
37.	a. x^8 b. $2f^5g^7$ c. $\frac{5x^3z^2}{6}$
38.	b. $2^3\cdot2^3\cdot2^3\cdot2^3$ c. $2^{10}\cdot2^{10}\cdot2^{10}\cdot2^{10}\cdot2^{10}$
39.	a. 6 b. 12 c. 50 d. $8X$
40.	a. 8 b. 10 c. 66 d. $4Y$
41.	$B^{10}\cdot B^{10}\cdot B^{10}\rightarrow B^{30}$
42.	a. $(2x)(2x)(2x)$ b. $(-3x)(-3x)(-3x)$
43.	a. $25x^8$ b. $-8y^3$ c. $\frac{z^8}{81}$
44.	$(-3x^{10})(-3x^{10})(-3x^{10})$
45.	a. $100x^{14}$ b. $-64y^6$ c. $625x^4y^8$
46.	a. $\frac{x^4}{81}$ b. $\frac{25}{x^4}$ c. $\frac{-8a^6}{b^9}$
47.	a. $8x^5$ b. $100x^4$ c. $216x^5$
48.	a. a^{MN} b. x^Ny^N c. $\frac{a^K}{b^K}$
49.	a. $x^{LN}y^{MN}$ b. $\frac{a^{GK}}{b^{HK}}$
50.	Power; When raising a power to a power, multiply the exponents.
51.	$2^{2x}+2^H=2^H+2^H=2\cdot2^H=2^{1+H}=2^W$
52.	a. $2(5x)^2\rightarrow2\cdot25x^2\rightarrow50x^2$ b. $-3(3y)^3\rightarrow-3\cdot27y^3\rightarrow-81y^3$
53.	a. $\frac{1}{2}$ b. $\frac{1}{10}$ c. $\frac{1}{5}$
54.	a. 1 b. 1 c. 1

